9.2 Graph $y = ax^2 + bx + c$

Before You graphed simple quadratic functions.

Now You will graph general quadratic functions.

Why? So you can investigate a cable's height, as in Example 4.

Key Vocabulary
- **minimum value**
- **maximum value**

COMMON CORE

CC.9-12.F.IF.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.*

You can use the properties below to graph any quadratic function. You will justify the formula for the axis of symmetry in Exercise 38 in this lesson.

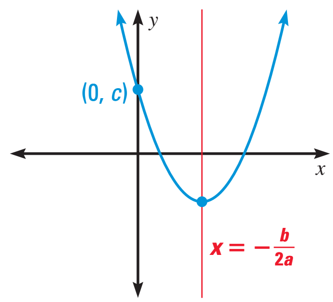
KEY CONCEPT *For Your Notebook*

Properties of the Graph of a Quadratic Function

The graph of $y = ax^2 + bx + c$ is a parabola that:

- opens up if $a > 0$ and opens down if $a < 0$.
- is narrower than the graph of $y = x^2$ if $|a| > 1$ and wider if $|a| < 1$.
- has an axis of symmetry of $x = -\frac{b}{2a}$.
- has a vertex with an x-coordinate of $-\frac{b}{2a}$.
- has a y-intercept of c. So, the point $(0, c)$ is on the parabola.

$y = ax^2 + bx + c, a > 0$



EXAMPLE 1 Find the axis of symmetry and the vertex

Consider the function $y = -2x^2 + 12x - 7$.

a. Find the axis of symmetry of the graph of the function.

b. Find the vertex of the graph of the function.

Solution

a. For the function $y = -2x^2 + 12x - 7$, $a = -2$ and $b = 12$.

$$x = -\frac{b}{2a} = -\frac{12}{2(-2)} = 3$$ **Substitute −2 for *a* and 12 for *b*. Then simplify.**

IDENTIFY THE VERTEX
Because the vertex lies on the axis of symmetry, x = 3, the x-coordinate of the vertex is 3.

b. The x-coordinate of the vertex is $-\frac{b}{2a}$, or 3.

To find the y-coordinate, substitute 3 for x in the function and find y.

$$y = -2(3)^2 + 12(3) - 7 = 11$$ **Substitute 3 for *x*. Then simplify.**

▶ The vertex is (3, 11).

EXAMPLE 2 Graph $y = ax^2 + bx + c$

Graph $y = 3x^2 - 6x + 2$.

STEP 1 **Determine** whether the parabola opens up or down. Because $a > 0$, the parabola opens up.

AVOID ERRORS
Be sure to include the negative sign before the fraction when calculating the axis of symmetry.

STEP 2 **Find** and draw the axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(3)} = 1$.

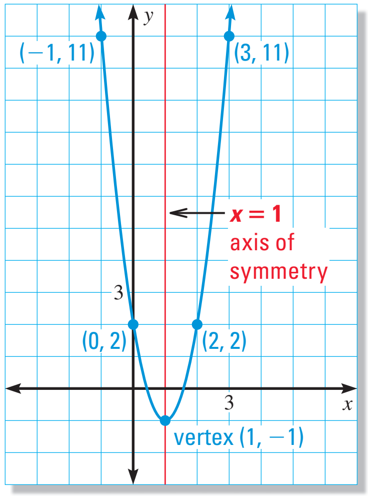
STEP 3 **Find** and plot the vertex.

The x-coordinate of the vertex is $-\frac{b}{2a}$, or 1.

To find the y-coordinate, substitute 1 for x in the function and simplify.

$y = 3(1)^2 - 6(1) + 2 = -1$

So, the vertex is $(1, -1)$.



STEP 4 **Plot** two points. Choose two x-values less than the x-coordinate of the vertex. Then find the corresponding y-values.

x	0	−1
y	2	11

REVIEW REFLECTIONS
For help with reflections, see p. SR14.

STEP 5 **Reflect** the points plotted in Step 4 in the axis of symmetry.

STEP 6 **Draw** a parabola through the plotted points.

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✓ GUIDED PRACTICE for Examples 1 and 2

1. Find the axis of symmetry and the vertex of the graph of the function $y = x^2 - 2x - 3$.

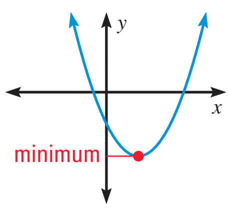
2. Graph the function $y = 3x^2 + 12x - 1$. Label the vertex and axis of symmetry.

KEY CONCEPT — For Your Notebook

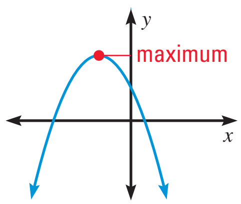
Minimum and Maximum Values

For $y = ax^2 + bx + c$, the y-coordinate of the vertex is the ______ of the function if $a > 0$ or the ______ of the function if $a < 0$.

$y = ax^2 + bx + c, a > 0$



$y = ax^2 + bx + c, a < 0$



EXAMPLE 3 Find the minimum or maximum value

Tell whether the function $f(x) = -3x^2 - 12x + 10$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

Solution

Because $a = -3$ and $-3 < 0$, the parabola opens down and the function has a maximum value. To find the maximum value, find the vertex.

$x = -\frac{b}{2a} = -\frac{-12}{2(-3)} = -2$ The x-coordinate is $-\frac{b}{2a}$.

$f(-2) = -3(-2)^2 - 12(-2) + 10 = 22$ Substitute -2 for x. Then simplify.

▶ The maximum value of the function is $f(-2) = 22$.

EXAMPLE 4 Find the minimum value of a function

SUSPENSION BRIDGES The suspension cables between the two towers of the Mackinac Bridge in Michigan form a parabola that can be modeled by the graph of $y = 0.000097x^2 - 0.37x + 549$ where x and y are measured in feet. What is the height of the cable above the water at its lowest point?

Solution

The lowest point of the cable is at the vertex of the parabola. Find the x-coordinate of the vertex. Use $a = 0.000097$ and $b = -0.37$.

$x = -\frac{b}{2a} = -\frac{-0.37}{2(0.000097)} \approx 1910$ Use a calculator.

Substitute 1910 for x in the equation to find the y-coordinate of the vertex.

$y \approx 0.000097(1910)^2 - 0.37(1910) + 549 \approx 196$

▶ The cable is about 196 feet above the water at its lowest point.

✓ GUIDED PRACTICE for Examples 3 and 4

3. Tell whether the function $f(x) = 6x^2 + 18x + 13$ has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.

4. **SUSPENSION BRIDGES** The cables between the two towers of the Tacoma Narrows Bridge form a parabola that can be modeled by the graph of the equation $y = 0.00014x^2 - 0.4x + 507$ where x and y are measured in feet. What is the height of the cable above the water at its lowest point? Round your answer to the nearest foot.

9.2 EXERCISES

HOMEWORK KEY

○ = See **WORKED-OUT SOLUTIONS** Exs. 9 and 41

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 12, 27, 37, 42, and 44

SKILL PRACTICE

1. **VOCABULARY** *Explain* how you can tell whether a quadratic function has a maximum value or minimum value without graphing the function.

2. ★ **WRITING** *Describe* the steps you would take to graph a quadratic function in standard form.

EXAMPLE 1 for Exs. 3–14

FINDING AXIS OF SYMMETRY AND VERTEX **Find the axis of symmetry and the vertex of the graph of the function.**

3. $y = 2x^2 - 8x + 6$
4. $y = x^2 - 6x + 11$
5. $y = -3x^2 + 24x - 22$
6. $y = -x^2 - 10x$
7. $y = 6x^2 + 6x$
8. $y = 4x^2 + 7$
9. $y = -\frac{2}{3}x^2 - 1$
10. $y = \frac{1}{2}x^2 + 8x - 9$
11. $y = -\frac{1}{4}x^2 + 3x - 2$

12. ★ **MULTIPLE CHOICE** What is the vertex of the graph of the function $y = -3x^2 + 18x - 13$?

Ⓐ $(-3, -94)$ Ⓑ $(-3, -14)$ Ⓒ $(3, -13)$ Ⓓ $(3, 14)$

ERROR ANALYSIS ***Describe*** **and correct the error in finding the axis of symmetry of the graph of the given function.**

13. $y = 2x^2 + 16x - 1$

$x = \frac{b}{2a} = \frac{16}{2(2)} = 4$

The axis of symmetry is $x = 4$.

14. $y = -\frac{3}{2}x^2 + 18x - 5$

$x = -\frac{b}{2a} = -\frac{18}{2\left(\frac{3}{2}\right)} = -6$

The axis of symmetry is $x = -6$.

EXAMPLE 2 for Exs. 15–27

GRAPHING QUADRATIC FUNCTIONS **Graph the function. Label the vertex and axis of symmetry.**

15. $y = x^2 + 6x + 2$
16. $y = x^2 + 4x + 8$
17. $y = 2x^2 + 7x + 21$
18. $y = 5x^2 + 10x - 3$
19. $y = 4x^2 + x - 32$
20. $y = -4x^2 + 4x + 8$
21. $y = -3x^2 - 2x - 5$
22. $y = -8x^2 - 12x + 1$
23. $y = -x^2 + \frac{1}{4}x + \frac{1}{2}$
24. $y = \frac{1}{3}x^2 + 6x - 9$
25. $y = -\frac{1}{2}x^2 + 6x + 3$
26. $y = -\frac{1}{4}x^2 - x + 1$

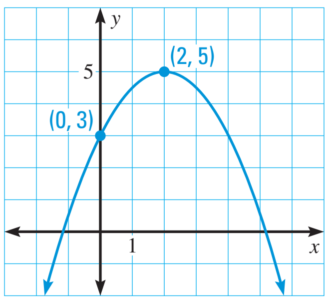
27. ★ **MULTIPLE CHOICE** Which function has the graph shown?

Ⓐ $y = -2x^2 + 8x + 3$

Ⓑ $y = -\frac{1}{2}x^2 + 2x + 3$

Ⓒ $y = \frac{1}{2}x^2 + 2x + 3$

Ⓓ $y = 2x^2 + 8x + 3$



EXAMPLE 3
for Exs. 28–36

MAXIMUM AND MINIMUM VALUES **Tell whether the function has a *minimum value* or a *maximum value*. Then find the minimum or maximum value.**

28. $f(x) = x^2 - 6$ **29.** $f(x) = -5x^2 + 7$ **30.** $f(x) = 4x^2 + 32x$

31. $f(x) = -3x^2 + 12x - 20$ **32.** $f(x) = x^2 + 7x + 8$ **33.** $f(x) = -2x^2 - x + 10$

34. $f(x) = \frac{1}{2}x^2 - 2x + 5$ **35.** $f(x) = -\frac{3}{8}x^2 + 9x$ **36.** $f(x) = \frac{1}{4}x^2 + 7x + 11$

37. ★ **WRITING** Compare the graph of $y = x^2 + 4x + 1$ with the graph of $y = x^2 - 4x + 1$.

38. REASONING Follow the steps below to justify the equation for the axis of symmetry for the graph of $y = ax^2 + bx + c$. Because the graph of $y = ax^2 + bx + c$ is a vertical translation of the graph of $y = ax^2 + bx$, the two graphs have the same axis of symmetry. Use the function $y = ax^2 + bx$ in place of $y = ax^2 + bx + c$.

a. Find the x-intercepts of the graph of $y = ax^2 + bx$. (You can do this by finding the zeros of the function $y = ax^2 + bx$ using factoring.)

b. Because a parabola is symmetric about its axis of symmetry, the axis of symmetry passes through a point halfway between the x-intercepts of the parabola. Find the x-coordinate of this point. What is an equation of the vertical line through this point?

39. CHALLENGE Write a function of the form $y = ax^2 + bx$ whose graph contains the points (1, 6) and (3, 6).

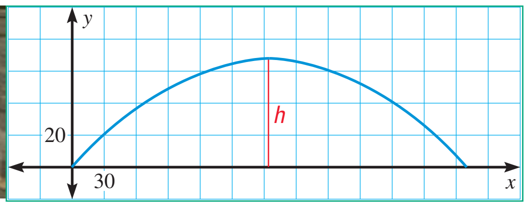
PROBLEM SOLVING

GRAPHING CALCULATOR **You may wish to use a graphing calculator to complete the following Problem Solving exercises.**

EXAMPLE 4
for Exs. 40–42

40. SPIDERS Fishing spiders can propel themselves across water and leap vertically from the surface of the water. During a vertical jump, the height of the body of the spider can be modeled by the function $y = -4500x^2 + 820x + 43$ where x is the duration (in seconds) of the jump and y is the height (in millimeters) of the spider above the surface of the water. After how many seconds does the spider's body reach its maximum height? What is the maximum height?

41. ARCHITECTURE The parabolic arches that support the roof of the Dallas Convention Center can be modeled by the graph of the equation $y = -0.0019x^2 + 0.71x$ where x and y are measured in feet. What is the height h at the highest point of the arch as shown in the diagram?



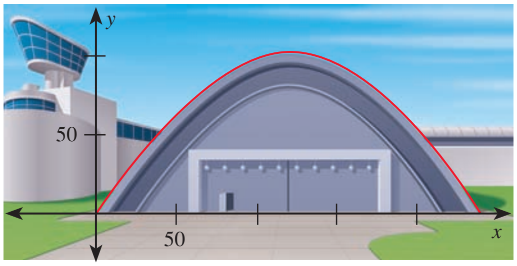
42. ★ **EXTENDED RESPONSE** Students are selling packages of flower bulbs to raise money for a class trip. Last year, when the students charged \$5 per package, they sold 150 packages. The students want to increase the cost per package. They estimate that they will lose 10 sales for each \$1 increase in the cost per package. The sales revenue R (in dollars) generated by selling the packages is given by the function $R = (5 + n)(150 - 10n)$ where n is the number of \$1 increases.

a. Write the function in standard form.

b. Find the maximum value of the function.

c. At what price should the packages be sold to generate the most sales revenue? *Explain* your reasoning.

43. **AIRCRAFT** An aircraft hangar is a large building where planes are stored. The opening of one airport hangar is a parabolic arch that can be modeled by the graph of the equation $y = -0.007x^2 + 1.7x$ where x and y are measured in feet. Graph the function. Use the graph to determine how wide the hangar is at its base.



44. ★ **SHORT RESPONSE** The casts of some Broadway shows go on tour, performing their shows in cities across the United States. For the period 1990–2001, the number of tickets sold S (in millions) for Broadway road tours can be modeled by the function $S = 332 + 132t - 10.4t^2$ where t is the number of years since 1990. Was the greatest number of tickets for Broadway road tours sold in 1995? *Explain*.

45. **CHALLENGE** During an archery competition, an archer shoots an arrow from 1.5 meters off of the ground. The arrow follows the parabolic path shown and hits the ground in front of the target 90 meters away. Use the y-intercept and the points on the graph to write an equation for the graph that models the path of the arrow.